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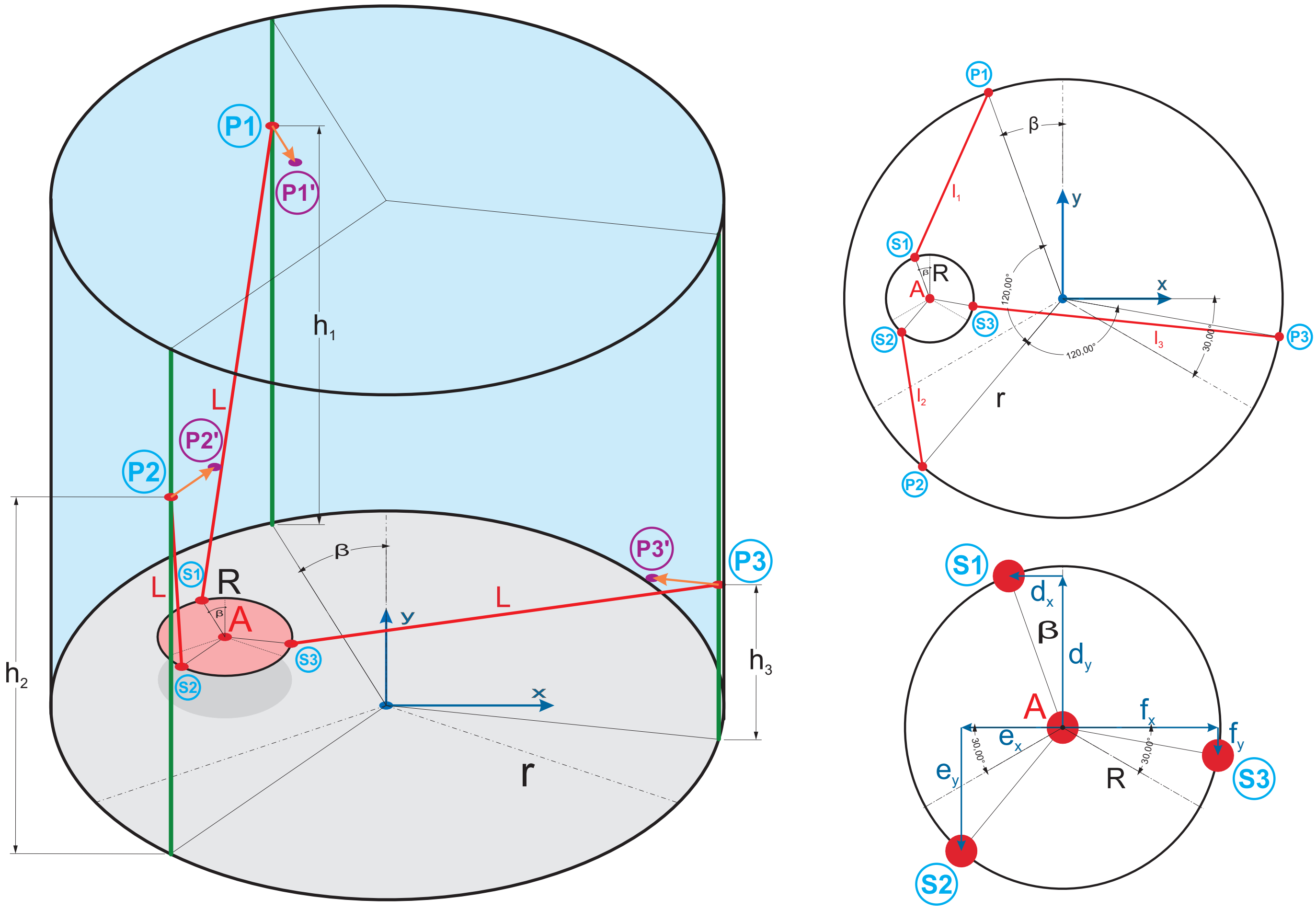
Teil I
Kinematische Berechnungen an einem Delta-Robot / Tri-Glide

1 Konstruktionszeichnung



2 Inverse Kinematik

2.1 Rechenweg

2.1.1 Ort

$$\begin{pmatrix} l_{1_x} \\ l_{1_y} \end{pmatrix} + \begin{pmatrix} -P1_x \\ -P1_y \end{pmatrix} + \begin{pmatrix} A_x \\ A_y \end{pmatrix} + \begin{pmatrix} d_x \\ d_y \end{pmatrix} = 0$$

$$\begin{pmatrix} l_{1_x} \\ l_{1_y} \end{pmatrix} = \begin{pmatrix} P1_x - A_x - d_x \\ P1_y - A_y - d_y \end{pmatrix}$$

$$l_1^2 = l_{1_x}^2 + l_{1_y}^2 = (P1_x - A_x - d_x)^2 + (P1_y - A_y - d_y)^2$$

$$h_1 = s_1 + A_z = \sqrt{L^2 - l_1^2} + A_z = \sqrt{L^2 - (P1_x - A_x - d_x)^2 - (P1_y - A_y - d_y)^2} + A_z$$

$$P1_x = -r \cdot sin(\beta)$$

$$P1_y = r \cdot cos(\beta)$$

$$d_x = -R \cdot sin(\beta)$$

$$d_y = R \cdot cos(\beta)$$

$$h_1 = \sqrt{L^2 - [-r \cdot sin(\beta) - A_x + R \cdot sin(\beta)]^2 - [r \cdot cos(\beta) - A_y - R \cdot cos(\beta)]^2} + A_z$$

$$h_1 = \sqrt{\underbrace{L^2}_{B_1} - [\underbrace{(R-r)sin(\beta)}_{B_2} - A_x]^2 - [\underbrace{(r-R)cos(\beta)}_{B_3} - A_y]^2} + A_z$$

$$h_1 = \sqrt{B_1 - (\underbrace{B_2 - A_x}_{F_1})^2 - (\underbrace{B_3 - A_y}_{F_2})^2} + A_z$$

$$h_1 = \underbrace{\sqrt{B_1 - F_1^2 - F_2^2}}_{F_3} + A_z$$

$$h_1 = F_3 + A_z$$

$$\begin{pmatrix} l_{2_x} \\ l_{2_y} \end{pmatrix} + \begin{pmatrix} -P2_x \\ -P2_y \end{pmatrix} + \begin{pmatrix} A_x \\ A_y \end{pmatrix} + \begin{pmatrix} e_x \\ e_y \end{pmatrix} = 0$$

$$\begin{pmatrix} l_{2_x} \\ l_{2_y} \end{pmatrix} = \begin{pmatrix} P2_x - A_x - e_x \\ P2_y - A_y - e_y \end{pmatrix}$$

$$l_2^2 = l_{2_x}^2 + l_{2_y}^2 = (P2_x - A_x - e_x)^2 + (P2_y - A_y - e_y)^2$$

$$h_2 = s_2 + A_z = \sqrt{L^2 - l_2^2} + A_z = \sqrt{L^2 - (P2_x - A_x - e_x)^2 - (P2_y - A_y - e_y)^2} + A_z$$

$$P2_x = -r \cdot cos(30^\circ + \beta)$$

$$P2_y = -r \cdot sin(30^\circ + \beta)$$

$$e_x = -R \cdot cos(30^\circ + \beta)$$

$$e_y = -R \cdot sin(30^\circ + \beta)$$

$$h_2 = \sqrt{L^2 - [-r \cdot cos(30^\circ + \beta) - A_x + R \cdot cos(30^\circ + \beta)]^2 - [-r \cdot sin(30^\circ + \beta) - A_y + R \cdot sin(30^\circ + \beta)]^2} + A_z$$

$$h_2 = \sqrt{L^2 - [\underbrace{(R-r)cos(30^\circ + \beta)}_{B_4} - A_x]^2 - [\underbrace{(R-r)sin(30^\circ + \beta)}_{B_5} - A_y]^2} + A_z$$

$$h_2 = \sqrt{B_1 - (\underbrace{B_4 - A_x}_{F_4})^2 - (\underbrace{B_5 - A_y}_{F_5})^2} + A_z$$

$$h_2 = \underbrace{\sqrt{B_1 - F_4^2 - F_5^2}}_{F_6} + A_z$$

$$h_2 = F_6 + A_z$$

$$\begin{pmatrix} l_{3_x} \\ l_{3_y} \end{pmatrix} + \begin{pmatrix} -P3_x \\ -P3_y \end{pmatrix} + \begin{pmatrix} A_x \\ A_y \end{pmatrix} + \begin{pmatrix} f_x \\ f_y \end{pmatrix} = 0$$

$$\begin{pmatrix} l_{3_x} \\ l_{3_y} \end{pmatrix} = \begin{pmatrix} P3_x - A_x - f_x \\ P3_y - A_y - f_y \end{pmatrix}$$

$$l_3^2 = l_{3_x}^2 + l_{3_y}^2 = (P3_x - A_x - f_x)^2 + (P3_y - A_y - f_y)^2$$

$$h_3 = s_3 + A_z = \sqrt{L^2 - l_3^2} + A_z = \sqrt{L^2 - (P3_x - A_x - f_x)^2 - (P3_y - A_y - f_y)^2} + A_z$$

$$P3_x = r \cdot cos(30^\circ - \beta)$$

$$P3_y = -r \cdot sin(30^\circ - \beta)$$

$$f_x = R \cdot cos(30^\circ - \beta)$$

$$f_y = -R \cdot sin(30^\circ - \beta)$$

$$h_3 = \sqrt{L^2 - [r \cdot cos(30^\circ - \beta) - A_x - R \cdot cos(30^\circ - \beta)]^2 - [-r \cdot sin(30^\circ - \beta) - A_y + R \cdot sin(30^\circ - \beta)]^2} + A_z$$

$$h_3 = \sqrt{L^2 - [\underbrace{(r - R)cos(30^\circ - \beta)}_{B_6} - A_x]^2 - [\underbrace{(R - r)sin(30^\circ - \beta)}_{B_7} - A_y]^2} + A_z$$

$$h_3 = \sqrt{B_1 - (\underbrace{B_6 - A_x}_{F_7})^2 - (\underbrace{B_7 - A_y}_{F_8})^2} + A_z$$

$$h_3 = \underbrace{\sqrt{B_1 - F_7^2 - F_8^2}}_{F_9} + A_z$$

$$h_3 = F_9 + A_z$$

2.1.2 Geschwindigkeit

$$\dot{h}_1 = \frac{d}{dt}\left[\sqrt{B_1 - (B_2 - A_x)^2 - (B_3 - A_y)^2} + A_z\right]$$

$$\dot{h}_1 = \frac{1}{2}\left[B_1 - (B_2 - A_x)^2 - (B_3 - A_y)^2\right]^{-\frac{1}{2}} \cdot [-2(B_2 - A_x)(-\dot{A}_x) - 2(B_3 - A_y)(-\dot{A}_y)] + \dot{A}_z$$

$$\dot{h}_1 = \frac{(B_2 - A_x)\dot{A}_x + (B_3 - A_y)\dot{A}_y}{\sqrt{B_1 - (B_2 - A_x)^2 - (B_3 - A_y)^2}} + \dot{A}_z$$

$$\dot{h}_1 = \frac{F_1\dot{A}_x + F_2\dot{A}_y}{F_3} + \dot{A}_z$$

$$\dot{h}_2 = \frac{d}{dt}\left[\sqrt{B_1 - (B_4 - A_x)^2 - (B_5 - A_y)^2} + A_z\right]$$

$$\dot{h}_2 = \frac{1}{2}\left[B_1 - (B_4 - A_x)^2 - (B_5 - A_y)^2\right]^{-\frac{1}{2}} \cdot [-2(B_4 - A_x)(-\dot{A}_x) - 2(B_5 - A_y)(-\dot{A}_y)] + \dot{A}_z$$

$$\dot{h}_2 = \frac{(B_4 - A_x)\dot{A}_x + (B_5 - A_y)\dot{A}_y}{\sqrt{B_1 - (B_4 - A_x)^2 - (B_5 - A_y)^2}} + \dot{A}_z$$

$$\dot{h}_2 = \frac{F_4\dot{A}_x + F_5\dot{A}_y}{F_6} + \dot{A}_z$$

$$\dot{h}_3 = \frac{d}{dt}\left[\sqrt{B_1 - (B_6 - A_x)^2 - (B_7 - A_y)^2} + A_z\right]$$

$$\dot{h}_3 = \frac{1}{2}\left[B_1 - (B_6 - A_x)^2 - (B_7 - A_y)^2\right]^{-\frac{1}{2}} \cdot [-2(B_6 - A_x)(-\dot{A}_x) - 2(B_7 - A_y)(-\dot{A}_y)] + \dot{A}_z$$

$$\dot{h}_3 = \frac{(B_6 - A_x)\dot{A}_x + (B_7 - A_y)\dot{A}_y}{\sqrt{B_1 - (B_6 - A_x)^2 - (B_7 - A_y)^2}} + \dot{A}_z$$

$$\dot{h}_3 = \frac{F_7\dot{A}_x + F_8\dot{A}_y}{F_9} + \dot{A}_z$$

2.2 Lösung

$$h_1 = F_3 + A_z$$
$$h_2 = F_6 + A_z$$
$$h_3 = F_9 + A_z$$

$$\dot{h}_1 = \frac{F_1\dot{A}_x + F_2\dot{A}_y}{F_3} + \dot{A}_z$$
$$\dot{h}_2 = \frac{F_4\dot{A}_x + F_5\dot{A}_y}{F_6} + \dot{A}_z$$
$$\dot{h}_3 = \frac{F_7\dot{A}_x + F_8\dot{A}_y}{F_9} + \dot{A}_z$$

$$B_1 = L^2$$
$$B_2 = (R-r)sin(\beta)$$
$$B_3 = (r-R)cos(\beta)$$
$$B_4 = (R-r)cos(30° + \beta)$$
$$B_5 = (R-r)sin(30° + \beta)$$
$$B_6 = (r-R)cos(30° - \beta)$$
$$B_7 = (R-r)sin(30° - \beta)$$

$$F_1 = B_2 - A_x$$
$$F_2 = B_3 - A_y$$
$$F_3 = \sqrt{B_1 - F_1^2 - F_2^2}$$
$$F_4 = B_4 - A_x$$
$$F_5 = B_5 - A_y$$
$$F_6 = \sqrt{B_1 - F_4^2 - F_5^2}$$
$$F_7 = B_6 - A_x$$
$$F_8 = B_7 - A_y$$
$$F_9 = \sqrt{B_1 - F_7^2 - F_8^2}$$

3 Vorwärts-Kinematik

3.1 Rechenweg

3.1.1 Ort

$$P1'_x = P1_x - d_x = -r \cdot sin(\beta) + R \cdot sin(\beta) = (R-r)sin(\beta)$$
$$P1'_y = P1_y - d_y = r \cdot cos(\beta) - R \cdot cos(\beta) = (r-R)cos(\beta)$$
$$P1'_z = h_1$$
$$L^2 = (A_x - P1'_x)^2 + (A_y - P1'_y)^2 + (A_z - P1'_z)^2$$
$$L^2 = A_x^2 - 2A_xP1'_x + {P1'_x}^2 + A_y^2 - 2A_yP1'_y + {P1'_y}^2 + A_z^2 - 2A_zP1'_z + {P1'_z}^2$$
$$L^2 = A_x^2 + A_y^2 + A_z^2 - 2A_xP1'_x - 2A_yP1'_y - 2A_zh_1 + \underbrace{{P1'_x}^2 + {P1'_y}^2}_{C_1} + \underbrace{h_1^2}_{D_1}$$
$$L^2 = A_x^2 + A_y^2 + A_z^2 - 2A_xP1'_x - 2A_yP1'_y - 2A_zh_1 + C_1 + D_1 \qquad (1)$$

$$P2'_x = P2_x - e_x = -r \cdot cos(30° + \beta) + R \cdot cos(30° + \beta) = (R-r)cos(30° + \beta)$$
$$P2'_y = P2_y - e_y = -r \cdot sin(30° + \beta) + R \cdot sin(30° + \beta) = (R-r)sin(30° + \beta)$$
$$P2'_z = h_2$$
$$L^2 = (A_x - P2'_x)^2 + (A_y - P2'_y)^2 + (A_z - P2'_z)^2$$
$$L^2 = A_x^2 - 2A_xP2'_x + {P2'_x}^2 + A_y^2 - 2A_yP2'_y + {P2'_y}^2 + A_z^2 - 2A_zP2'_z + {P2'_z}^2$$
$$L^2 = A_x^2 + A_y^2 + A_z^2 - 2A_xP2'_x - 2A_yP2'_y - 2A_zh_2 + \underbrace{{P2'_x}^2 + {P2'_y}^2}_{C_2} + \underbrace{h_2^2}_{D_2}$$
$$L^2 = A_x^2 + A_y^2 + A_z^2 - 2A_xP2'_x - 2A_yP2'_y - 2A_zh_2 + C_2 + D_2 \qquad (2)$$

$$P3'_x = P3_x - f_x = r \cdot cos(30° - \beta) - R \cdot cos(30° - \beta) = (r-R)cos(30° - \beta)$$
$$P3'_y = P3_y - f_y = -r \cdot sin(30° - \beta) + R \cdot sin(30° - \beta) = (R-r)sin(30° - \beta)$$
$$P3'_z = h_3$$
$$L^2 = (A_x - P3'_x)^2 + (A_y - P3'_y)^2 + (A_z - P3'_z)^2$$
$$L^2 = A_x^2 - 2A_xP3'_x + {P3'_x}^2 + A_y^2 - 2A_yP3'_y + {P3'_y}^2 + A_z^2 - 2A_zP3'_z + {P3'_z}^2$$
$$L^2 = A_x^2 + A_y^2 + A_z^2 - 2A_xP3'_x - 2A_yP3'_y - 2A_zh_3 + \underbrace{{P3'_x}^2 + {P3'_y}^2}_{C_3} + \underbrace{h_3^2}_{D_3}$$
$$L^2 = A_x^2 + A_y^2 + A_z^2 - 2A_xP3'_x - 2A_yP3'_y - 2A_zh_3 + C_3 + D_3 \qquad (3)$$

$(1)-(2):$

$$0=-2A_xP1'_x+2A_xP2'_x-2A_yP1'_y+2A_yP2'_y-2A_zh_1+2A_zh_2+C_1+D_1-C_2-D_2$$

$$0=\underbrace{(P2'_x-P1'_x)}_{C_4}A_x+\underbrace{(P2'_y-P1'_y)}_{C_5}A_y+\underbrace{(h_2-h_1)}_{D_4}A_z+\frac{1}{2}(\underbrace{C_1-C_2}_{C_6}+\underbrace{D_1-D_2}_{D_5})$$

$$0=C_4A_x+C_5A_y+D_4A_z+\frac{1}{2}(C_6+D_5)$$

$(1)-(3):$

$$0=-2A_xP1'_x+2A_xP3'_x-2A_yP1'_y+2A_yP3'_y-2A_zh_1+2A_zh_3+C_1+D_1-C_3-D_3$$

$$0=\underbrace{(P3'_x-P1'_x)}_{C_7}A_x+\underbrace{(P3'_y-P1'_y)}_{C_8}A_y+\underbrace{(h_3-h_1)}_{D_6}A_z+\frac{1}{2}(\underbrace{C_1-C_3}_{C_9}+\underbrace{D_1-D_3}_{D_7})$$

$$0=C_7A_x+C_8A_y+D_6A_z+\frac{1}{2}(C_9+D_7)$$

$$C_4A_x=-C_5A_y-D_4A_z-\frac{1}{2}(C_6+D_5)$$

$$A_x=-\frac{C_5}{C_4}A_y-\frac{D_4}{C_4}A_z-\frac{C_6+D_5}{2C_4}$$

$$0=-\frac{C_5C_7}{C_4}A_y-\frac{C_7D_4}{C_4}A_z-\frac{C_7(C_6+D_5)}{2C_4}+C_8A_y+D_6A_z+\frac{1}{2}(C_9+D_7)$$

$$0=\frac{C_4C_8-C_5C_7}{C_4}A_y+\frac{C_4D_6-D_4C_7}{C_4}A_z+\frac{C_4(C_9+D_7)-C_7(C_6+D_5)}{2C_4}$$

$$A_y=\frac{C_7D_4-C_4D_6}{C_4C_8-C_5C_7}A_z+\frac{C_7(C_6+D_5)-C_4(C_9+D_7)}{2\underbrace{[C_4C_8-C_5C_7]}_{C_{10}}}$$

$$A_y=\underbrace{\frac{C_7D_4-C_4D_6}{C_{10}}}_{D_8}A_z+\underbrace{\frac{C_7(C_6+D_5)-C_4(C_9+D_7)}{2C_{10}}}_{D_9}$$

$$A_y=D_8A_z+D_9$$

$$C_8A_y=-C_7A_x-D_6A_z-\frac{1}{2}(C_9+D_7)$$

$$A_y=-\frac{C_7}{C_8}A_x-\frac{D_6}{C_8}A_z-\frac{C_9+D_7}{2C_8}$$

$$0=C_4A_x-\frac{C_5C_7}{C_8}A_x-\frac{C_5D_6}{C_8}A_z-\frac{C_5(C_9+D_7)}{2C_8}+D_4A_z+\frac{1}{2}(C_6+D_5)$$

$$0=\frac{C_4C_8-C_5C_7}{C_8}A_x+\frac{C_8D_4-C_5D_6}{C_8}A_z+\frac{C_8(C_6+D_5)-C_5(C_9+D_7)}{2C_8}$$

$$A_x=\frac{C_5D_6-C_8D_4}{C_4C_8-C_5C_7}A_z+\frac{C_5(C_9+D_7)-C_8(C_6+D_5)}{2(C_4C_8-C_5C_7)}$$

$$A_x=\underbrace{\frac{C_5D_6-C_8D_4}{C_{10}}}_{D_{10}}A_z+\underbrace{\frac{C_5(C_9+D_7)-C_8(C_6+D_5)}{2C_{10}}}_{D_{11}}$$

$$A_x=D_{10}A_z+D_{11}$$

$(1):$

$$L^2=A_x^2+A_y^2+A_z^2-2A_xP1'_x-2A_yP1'_y-2A_zh_1+C_1+D_1$$

$$0=(D_{10}A_z+D_{11})^2+(D_8A_z+D_9)^2+A_z^2-2P1'_x(D_{10}A_z+D_{11})-2P1'_y(D_8A_z+D_9)-2A_zh_1+D_1+\underbrace{C_1-L^2}_{C_{11}}$$

$$0=D_{10}^2A_z^2+2D_{10}D_{11}A_z+D_{11}^2+D_8^2A_z^2+2D_8D_9A_z+D_9^2+A_z^2-2P1'_xD_{10}A_z-2P1'_xD_{11}-2P1'_yD_8A_z-2P1'_yD_9-2A_zh_1+D_1+C_{11}$$

$$0=(D_8^2+D_{10}^2+1)A_z^2+2(D_{10}D_{11}+D_8D_9-P1'_xD_{10}-P1'_yD_8-h_1)A_z+D_{11}^2+D_9^2-2(P1'_xD_{11}+P1'_yD_9)+D_1+C_{11}$$

$$0 = \underbrace{(D_8^2 + D_{10}^2 + 1)}_{D_{12}} A_z^2 + \underbrace{2[D_8(D_9 - P1'_y) + D_{10}(D_{11} - P1'_x) - h_1]}_{D_{13}} A_z + \underbrace{D_9^2 + D_{11}^2 - 2(P1'_x D_{11} + P1'_y D_9) + D_1 + C_{11}}_{D_{14}}$$

$$0 = D_{12}A_z^2 + D_{13}A_z + D_{14}$$

$$A_z = -\frac{D_{13} + \sqrt{D_{13}^2 - 4D_{12}D_{14}}}{2D_{12}}$$

3.1.2 Geschwindigkeit

$$\dot{A}_z = \frac{d}{dt}\left[-\frac{D_{13} + \sqrt{D_{13}^2 - 4D_{12}D_{14}}}{2D_{12}}\right]$$

$$\dot{A}_z = -\frac{2D_{12}\left[\dot{D}_{13} + \frac{1}{2}(D_{13}^2 - 4D_{12}D_{14})^{-\frac{1}{2}} \cdot (2D_{13}\dot{D}_{13} - 4(D_{12}\dot{D}_{14} + \dot{D}_{12}D_{14}))\right] - \left[D_{13} + \overbrace{\sqrt{D_{13}^2 - 4D_{12}D_{14}}}^{D_{15}}\right] \cdot 2\dot{D}_{12}}{4D_{12}^2}$$

$$\dot{A}_z = -\frac{D_{12}\left[\dot{D}_{13} + \frac{D_{13}\dot{D}_{13} - 2(D_{12}\dot{D}_{14} + \dot{D}_{12}D_{14})}{D_{15}} - \dot{D}_{12}\left(D_{13} + D_{15}\right)\right]}{2D_{12}^2}$$

$$\dot{A}_y = \frac{d}{dt}[D_8A_z + D_9] = D_8\dot{A}_z + \dot{D}_8A_z + \dot{D}_9$$

$$\dot{A}_x = \frac{d}{dt}[D_{10}A_z + D_{11}] = D_{10}\dot{A}_z + \dot{D}_{10}A_z + \dot{D}_{11}$$

$$\dot{D}_1 = \frac{d}{dt}\left[h_1^2\right] = 2h_1\dot{h}_1$$

$$\dot{D}_2 = \frac{d}{dt}\left[h_2^2\right] = 2h_2\dot{h}_2$$

$$\dot{D}_3 = \frac{d}{dt}\left[h_3^2\right] = 2h_3\dot{h}_3$$

$$\dot{D}_4 = \frac{d}{dt}[h_2 - h_1] = \dot{h}_2 - \dot{h}_1$$

$$\dot{D}_5 = \frac{d}{dt}[D_1 - D_2] = \dot{D}_1 - \dot{D}_2$$

$$\dot{D}_6 = \frac{d}{dt}[h_3 - h_1] = \dot{h}_3 - \dot{h}_1$$

$$\dot{D}_7 = \frac{d}{dt}[D_1 - D_3] = \dot{D}_1 - \dot{D}_3$$

$$\dot{D}_8 = \frac{d}{dt}\left[\frac{C_7D_4 - C_4D_6}{C_{10}}\right] = \frac{C_7\dot{D}_4 - C_4\dot{D}_6}{C_{10}}$$

$$\dot{D}_9 = \frac{d}{dt}\left[\frac{C_7(C_6 + D_5) - C_4(C_9 + D_7)}{2C_{10}}\right] = \frac{C_7\dot{D}_5 - C_4\dot{D}_7}{2C_{10}}$$

$$\dot{D}_{10} = \frac{d}{dt}\left[\frac{C_5D_6 - C_8D_4}{C_{10}}\right] = \frac{C_5\dot{D}_6 - C_8\dot{D}_4}{C_{10}}$$

$$\dot{D}_{11} = \frac{d}{dt}\left[\frac{C_5(C_9 + D_7) - C_8(C_6 + D_5)}{2C_{10}}\right] = \frac{C_5\dot{D}_7 - C_8\dot{D}_5}{2C_{10}}$$

$$\dot{D}_{12} = \frac{d}{dt}\left[D_8^2 + D_{10}^2 + 1\right] = 2D_8\dot{D}_8 + 2D_{10}\dot{D}_{10} = 2[D_8\dot{D}_8 + D_{10}\dot{D}_{10}]$$

$$\dot{D}_{13} = \frac{d}{dt}\left[2[D_8(D_9 - P1'_y) + D_{10}(D_{11} - P1'_x) - h_1]\right] = 2[D_8\dot{D}_9 + \dot{D}_8D_9 - P1'_y\dot{D}_8 + D_{10}\dot{D}_{11} + \dot{D}_{10}D_{11} - P1'_x\dot{D}_{10} - \dot{h}_1]$$

$$\dot{D}_{13} = 2[D_8\dot{D}_9 + \dot{D}_8(D_9 - P1'_y) + D_{10}\dot{D}_{11} + \dot{D}_{10}(D_{11} - P1'_x) - \dot{h}_1]$$

$$\dot{D}_{14} = \frac{d}{dt}\left[D_9^2 + D_{11}^2 - 2(P1'_xD_{11} + P1'_yD_9) + D_1 + C_{11}\right] = 2D_9\dot{D}_9 + 2D_{11}\dot{D}_{11} - 2(P1'_x\dot{D}_{11} + P1'_y\dot{D}_9) + \dot{D}_1$$

$$\dot{D}_{14} = 2[D_9\dot{D}_9 + D_{11}\dot{D}_{11} - P1'_x\dot{D}_{11} - P1'_y\dot{D}_9] + \dot{D}_1 = 2[\dot{D}_9(D_9 - P1'_y) + \dot{D}_{11}(D_{11} - P1'_x)] + \dot{D}_1$$

3.2 Lösung

$$A_z = -\frac{D_{13} + \sqrt{D_{13}^2 - 4D_{12}D_{14}}}{2D_{12}}$$
$$A_y = D_8 A_z + D_9$$
$$A_x = D_{10} A_z + D_{11}$$

$$\dot{A}_z = -\frac{D_{12}\left[\dot{D}_{13} + \frac{D_{13}\dot{D}_{13} - 2(D_{12}\dot{D}_{14} + \dot{D}_{12}D_{14})}{D_{15}} - \dot{D}_{12}\left(D_{13} + D_{15}\right)\right]}{2D_{12}^2}$$
$$\dot{A}_y = D_8\dot{A}_z + \dot{D}_8 A_z + \dot{D}_9$$
$$\dot{A}_x = D_{10}\dot{A}_z + \dot{D}_{10} A_z + \dot{D}_{11}$$

$$C_1 = P1_x'^2 + P1_y'^2$$
$$C_2 = P2_x'^2 + P2_y'^2$$
$$C_3 = P3_x'^2 + P3_y'^2$$
$$C_4 = P2_x' - P1_x'$$
$$C_5 = P2_y' - P1_y'$$
$$C_6 = C_1 - C_2$$
$$C_7 = P3_x' - P1_x'$$
$$C_8 = P3_y' - P1_y'$$
$$C_9 = C_1 - C_3$$
$$C_{10} = C_4 C_8 - C_5 C_7$$
$$C_{11} = C_1 - L^2$$

$$P1_x' = (R - r)sin(\beta)$$
$$P1_y' = (r - R)cos(\beta)$$
$$P2_x' = (R - r)cos(30^\circ + \beta)$$
$$P2_y' = (R - r)sin(30^\circ + \beta)$$
$$P3_x' = (r - R)cos(30^\circ - \beta)$$
$$P3_y' = (R - r)sin(30^\circ - \beta)$$

$$D_1 = h_1^2$$
$$D_2 = h_2^2$$
$$D_3 = h_3^2$$
$$D_4 = h_2 - h_1$$
$$D_5 = D_1 - D_2$$
$$D_6 = h_3 - h_1$$
$$D_7 = D_1 - D_3$$
$$D_8 = \frac{C_7 D_4 - C_4 D_6}{C_{10}}$$
$$D_9 = \frac{C_7(C_6 + D_5) - C_4(C_9 + D_7)}{2C_{10}}$$
$$D_{10} = \frac{C_5 D_6 - C_8 D_4}{C_{10}}$$
$$D_{11} = \frac{C_5(C_9 + D_7) - C_8(C_6 + D_5)}{2C_{10}}$$
$$D_{12} = D_8^2 + D_{10}^2 + 1$$
$$D_{13} = 2[D_8(D_9 - P1_y') + D_{10}(D_{11} - P1_x') - h_1]$$
$$D_{14} = D_9^2 + D_{11}^2 - 2(P1_x' D_{11} + P1_y' D_9) + D_1 + C_{11}$$
$$D_{15} = \sqrt{D_{13}^2 - 4D_{12}D_{14}}$$

$$\dot{D}_1 = 2h_1\dot{h}_1$$
$$\dot{D}_2 = 2h_2\dot{h}_2$$
$$\dot{D}_3 = 2h_3\dot{h}_3$$
$$\dot{D}_4 = \dot{h}_2 - \dot{h}_1$$
$$\dot{D}_5 = \dot{D}_1 - \dot{D}_2$$
$$\dot{D}_6 = \dot{h}_3 - \dot{h}_1$$
$$\dot{D}_7 = \dot{D}_1 - \dot{D}_3$$
$$\dot{D}_8 = \frac{C_7\dot{D}_4 - C_4\dot{D}_6}{C_{10}}$$
$$\dot{D}_9 = \frac{C_7\dot{D}_5 - C_4\dot{D}_7}{2C_{10}}$$
$$\dot{D}_{10} = \frac{C_5\dot{D}_6 - C_8\dot{D}_4}{C_{10}}$$
$$\dot{D}_{11} = \frac{C_5\dot{D}_7 - C_8\dot{D}_5}{2C_{10}}$$
$$\dot{D}_{12} = 2[D_8\dot{D}_8 + D_{10}\dot{D}_{10}]$$
$$\dot{D}_{13} = 2[D_8\dot{D}_9 + \dot{D}_8(D_9 - P1_y') + D_{10}\dot{D}_{11} + \dot{D}_{10}(D_{11} - P1_x') - \dot{h}_1]$$
$$\dot{D}_{14} = 2[\dot{D}_9(D_9 - P1_y') + \dot{D}_{11}(D_{11} - P1_x')] + \dot{D}_1$$

